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NONLINEAR VIBRATIONS OF A BODY MOUNTED ON VISCOELASTIC OSCILLATING SUPPORTS

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Abstract. *The paper considers nonlinear vibrations of a solid body mounted on viscoelastic supports. The equations of motion of the system are derived from the Lagrange equations of the second kind for systems with a finite number of degrees of freedom. A method for solving the problem is developed and numerical results are obtained, the influence of nonlinearity on the displacement amplitudes is estimated.*

Keywords*: nonlinear oscillations, solid, oscillation, Lagrange equations, viscoelastic support, degree of freedom.*

1. Introduction

Issues related to the dynamic damping of mechanical systems' vibrations are widely used in the dynamics of machines [1, p.254; 2, p. 129]. Passive methods of vibration reduction are especially effective for objects subject to sustained periodic external influences. There are known developments related to the creation of complex systems for reducing vibrations with several perturbations [3, p.187; 4, p.111]. Of particular interest are theoretical approaches to solving problems of vibration reduction based on structural methods of mathematical modeling [5,p. 173; 6,p. 138], in which a mechanical oscillatory system is interpreted as some kind of dynamic automatic control system. In this case, the structural scheme of the system acts as a structural analogue of the original mathematical model obtained in the form of a system of differential equations [7, p.174; 8, p.307]. The object of protection against vibration effects in the structural scheme of a vibration-proof system can be distinguished by structural transformations as an integrating link of the second order when bringing the structure of a mechanical oscillatory system (or vibration-proof system) to a scheme consisting of a protection object and a negative feedback circuit covering it. In this case, it becomes possible to determine the modes of dynamic damping, based on the properties of the transfer functions of feedback circuits [9, p.136; 10, p.178]. In the physical sense, the negative feedback relative to the object of protection in the structural model generically reflects the elastic properties of the vibration protection system or the reduced stiffness, depending on the frequency of external action.

2. Methods

2.1. Problem statement and solution methods

Consider the vibrations of a solid body mounted on viscoelastic supports. The mechanical system shown in Fig.1, i.e., a rigid body 1, is mounted on a rigid plate 2 with the help of viscoelastic supports (springs) 3. We determine the amplitude-frequency characteristics of various points of the body for a given harmonic oscillation law of the base plate. The equations of motion of the system are obtained from the Lagrange equations Type II for a system with elastic supports by replacing the stiffness of the supports with integral operators [6,p.245]:

$$\left.\begin{aligned}
&m\ddot{x}_c + c_x x_c - cl\varphi - \varepsilon_1\left(c_x \int_{-\infty}^{t} R_x(t-\tau)x_c(\tau)\,d\tau + c_x l \int_{\infty}^{t} R_x(t-\tau)\varphi(\tau)\,d\tau \right) = \\
&= A\sin\nu t + \varepsilon \frac{c_x l}{6}\left(\varphi^3 - \int_{-\infty}^{t} R_x(t-\tau)\varphi^3(\tau)\,d\tau \right), \\
&I\ddot{\varphi} - c_x l x_c - \left(c_\varphi + c_x l^2\right)\varphi + \varepsilon_1\left(c_x l \int_{-\infty}^{t} R_x(t-\tau)x_c(\tau)\,d\tau - \int_{-\infty}^{t} \left[c_\varphi R_\varphi(t-\tau) + c_x l^2 R_\varphi(t-\tau) \right]\varphi(\tau)\,d\tau \right) = \\
&= \varepsilon\left\{ -\frac{c_x l}{2}\left(\varphi^2 x_c - \int_{-\infty}^{t} R_x(t-\tau)\varphi^2(\tau)x_c(\tau)\,d\tau \right) + \frac{8}{6}\left(\varphi^3 - \int_{-\infty}^{t} R_x(t-\tau)\varphi^3(\tau)\,d\tau \right) \right\}.
\end{aligned}\right\} \quad (1)$$

where c_x, c_φ — stiffness of the supports in the direction of the two generalized coordinates***; I*** — moment of inertia, R_x and R_φ — kernels relaxation in directions x and φ; m — body weight.

The generating system (1) at $\varepsilon = 0,\ \ \varepsilon_1 = 0$ has frequencies

$$\omega_{1,2}^2 = \frac{1}{2}\left[\frac{c_x l^2 + c_\varphi}{I} + \frac{c_x}{m} \right] \pm \sqrt{\frac{1}{4}\left[\frac{c_x l^2 + c_\varphi}{I} + \frac{c_x}{m} \right]^2 - \frac{c_x c_\varphi}{mI}}. \quad (2)$$

Let's make a transition to normal coordinates for (1), presenting the solution by embedding in normal forms:

$$x = a_{11} f_1 + a_{12} f_2, \quad \varphi = a_{21} f_1 + a_{22} f_2. \quad (3)$$

We satisfy the oscillation equations (1) at $\varepsilon = 0,\ \ \varepsilon_1 = 0$ with the functions

$$\left.\begin{aligned}
x = a_{11}\sin(\omega_1 t + \alpha_1), \quad x = a_{21}\sin(\omega_2 t + \alpha_2), \\
\varphi = a_{21}\sin(\omega_1 t + \alpha_1), \quad \varphi = a_{22}\sin(\omega_2 t + \alpha_2),
\end{aligned}\right\} \quad (4)$$

where $a_{11}, a_{12}, a_{21}, a_{22}$ – the desired amplitudes.

Substituting (4) into (1), we obtain auxiliary relations

$$\left.\begin{aligned}&-ma_{11}\omega_1^2+c_xla_{11}-c_xla_{21}=0,\\&-Ia_{21}\omega_1^2-c_xla_{11}+\left(c_xl^2+c_c\right)a_{21}=0,\\&-ma_{12}\omega_2^2+c_xla_{12}-c_xla_{22}=0,\\&-Ia_{22}\omega_2^2-c_xla_{12}+\left(c_xl^2+c_c\right)a_{22}=0,\end{aligned}\right\}\qquad(5)$$

allowing to determine the amplitudes of $a_{11},\ a_{12},\ a_{21},\ a_{22}-$ in the form of

$$\left.\begin{aligned}&a_{21}=\frac{m\omega_1^2+c_xx_c+c_xl}{c_xl^2+c_xl+c_\varphi-I\omega_1^2},\\&a_{22}=-\frac{m\omega_2^2+c_xl-c_xl}{c_xl^2-c_x+c_\varphi-I\omega_2^2},\\&a_{11}=a_{12}=1.\end{aligned}\right\}\qquad(6)$$

Thus, the system (1), taking into account (3) and (5), takes the form

$$\ddot{f}_1+\Omega_2\left[f_1-\int_{-\infty}^{t}R_1(t-\tau)f_1(\tau)d\tau\right]=P\sin\nu t+F_1(f_1,f_2),\qquad(7)$$

$$\ddot{f}_2+\Omega^2\left[f_2-\int_{-\infty}^{t}R_2(t-\tau)f_2(\tau)d\tau\right]=F_2(f_1,f_2),\qquad(8)$$

Periodic solutions (7) and (8), according to [7, p.174; 8, p.482], we express as follows:

$$f_1=f_{1,0}+\sum_{n=1}^{\infty}\left[f_{1,n}-f_{1,n(n-1)}\right],\qquad(9)$$

$$f_2=f_{2,0}+\sum_{n=1}^{\infty}\left[f_{2,n}-f_{2,n(n-1)}\right],\qquad(10)$$

Substituting (9) and (10), respectively, into equations (7) and (8), we obtain a system of equations [11,p.169]

$$\left.\begin{aligned}&\ddot{f}_{1,n}+\Omega^2\left[f_{1,n}-\int_{-\infty}^{t}R_1(t-\tau)f_{1,n}(\tau)d\tau\right]=P\sin\nu t+F_1\left(f_{1,(n-1)},f_{2,(n-1)}\right),\\&\ddot{f}_{2,n}+\Omega^2\left[f_{2,n}-\int_{-\infty}^{t}R_2(t-\tau)f_{2,n}(\tau)d\tau\right]=F_2\left(f_{1,(n-1)},f_{2,(n-1)}\right).\end{aligned}\right\}\qquad(11)$$

Let the functions $F_1(f_1,\ f_2)$ and $F_2(f_1,\ f_2)$ be representable as uniformly convergent Fourier series:

$$\left.\begin{aligned}&F_1(f_1,\ f_2)=\sum_{k_1=0}^{\infty}\psi_{k_1}\sin(\mu_kt+\nu_k),\\&F_2(f_1,\ f_2)=\sum_{k_2=0}^{\infty}\psi_{k_2}\sin(\mu_kt+\nu_k),\end{aligned}\right\}\qquad(12)$$

The solution of equations (7), (8) under the conditions of $F_1(f_1, f_2)=0$ and $F_2(f_1, f_2)=0$ will be:

$$\left.\begin{aligned} f_1(t)&=\sum_{k_1=0}^{\infty} c_{k_1}\sin(\mu_k t+\beta_k),\\ f_2(t)&=\sum_{k_2=0}^{\infty} c_{k_2}\sin(\mu_k t+\beta_k),\end{aligned}\right\} \quad (13)$$

where

$$\left.\begin{aligned} c_{k_1}&=\frac{F_{k_1}}{\left[\Omega^4 R_{sk}+\left(\Omega^2\left(1-R'_{ck}\right)-\mu_k^2\right)^2\right]^{1/2}},\\ c_{k_2}&=\frac{F_{k_2}}{\left[\Omega^4 R_{sk}+\left(\Omega^2\left(1-R'_{ck}\right)-\mu_k^2\right)^2\right]^{1/2}},\end{aligned}\right\} \quad (14)$$

$$R'_{sk}=\int_0^{\infty} R(\tau)\sin\mu_k\tau d\tau; \qquad R'_{ck}=\int_0^{\infty} R(\tau)\cos\mu_k\tau d\tau; \qquad (*)$$

where R'_{sk} and R'_{ck} are defined by expressions (*); β_k constants satisfy the equation

$$tg(\gamma_k-\beta_k)=\frac{\Omega^2 R_{sk}}{\Omega^2(1-R_{ck})-\mu_k^2}. \qquad (15)$$

The series (13) converges absolutely and uniformly under the condition of $0<R_c<1$. Function (13) does not contain arbitrary constants and characterizes forced oscillations in the presence of periodic disturbances (12). Consider nonlinear integro-differential equations (7-8), presenting them in the following form*:*

$$\left.\begin{aligned} \ddot{f}_{1n}+\Omega_1^2\left[f_{1n}-\int_{-\infty}^{t} R_1(t-\tau) f_{1n}(\tau)d\tau\right]&=P_1\sin\nu t+\mu F_{1n}(f_1, f_2),\\ \ddot{f}_{2n}+\Omega_2^2\left[f_{2n}-\int_{-\infty}^{t} R_2(t-\tau) f_{2n}(\tau)d\tau\right]&=P_2\sin\nu t+\mu F_{2n}(f_1, f_2);\end{aligned}\right\} \quad (16)$$

here f_{1n} and f_{2n} - the desired functions; $\Omega-$ known constant; $F_{1n}(f_1, f_2)$ and $F_{2n}(f_1, f_2)-$ known functions; $\mu-$ dimensionless positive parameter.

We assume that the kernel of R satisfies the condition [12, p.3; 13, p.5]

$$0<\int_0^{\infty} R(\tau)d\tau<1, \qquad (17)$$

and the functions $F_{1n}(f_1, f_2)$ and $F_{2n}(f_1, f_2)$ are representable by an infinite series (12).

We will look for the periodic solution of equations (16) in the form of infinite series:

$$\left.\begin{aligned} f_{1n}(t) &= f_{10}(t) + \sum_{k=0}^{\infty}\left(f_{1k}(t) - f_{1(k-1)}(t)\right), \\ f_{2n}(t) &= f_{20}(t) + \sum_{k=0}^{\infty}\left(f_{2k}(t) - f_{2(k-1)}(t)\right), \end{aligned}\right\} \qquad (18)$$

moreover, the terms of this series are solutions of linear equations

$$\left.\begin{aligned} &\ddot{f}_{10} + \Omega^2\left[f_{1,0} - \int_{-\infty}^{t} R_1(t-\tau) f_{10}(\tau) d\tau \right] = P_1 \sin \nu t, \\ &\ddot{f}_{20} + \Omega^2\left[f_{2,0} - \int_{-\infty}^{t} R_2(t-\tau) f_{20}(\tau) d\tau \right] = P_2 \sin \nu t, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\ddot{f}_{1k} + \Omega^2\left[f_{1,k} - \int_{-\infty}^{t} R_{1k}(t-\tau) f_{1k}(\tau) d\tau \right] = P_1 \sin \nu t + \mu F_{1k}\left(f_{1(k-1)}, f_{2(k-1)}, t\right), \\ &\ddot{f}_{2k} + \Omega^2\left[f_{2,k} - \int_{-\infty}^{t} R_{2k}(t-\tau) f_{2k}(\tau) d\tau \right] = P_2 \sin \nu t + \mu F_{2k}\left(f_{1(k-1)}, f_{2(k-1)}, t\right). \end{aligned}\right\} \qquad (19)$$

Consequently, linear equations (19) are the result of applying the method of successive approximations to the original nonlinear equation (16) [10,p. 339]. Since the right-hand sides of equation (19) are periodic functions of time, we find its solution in the form (13), while the right-hand sides (19) should be decomposed into a Fourier series.

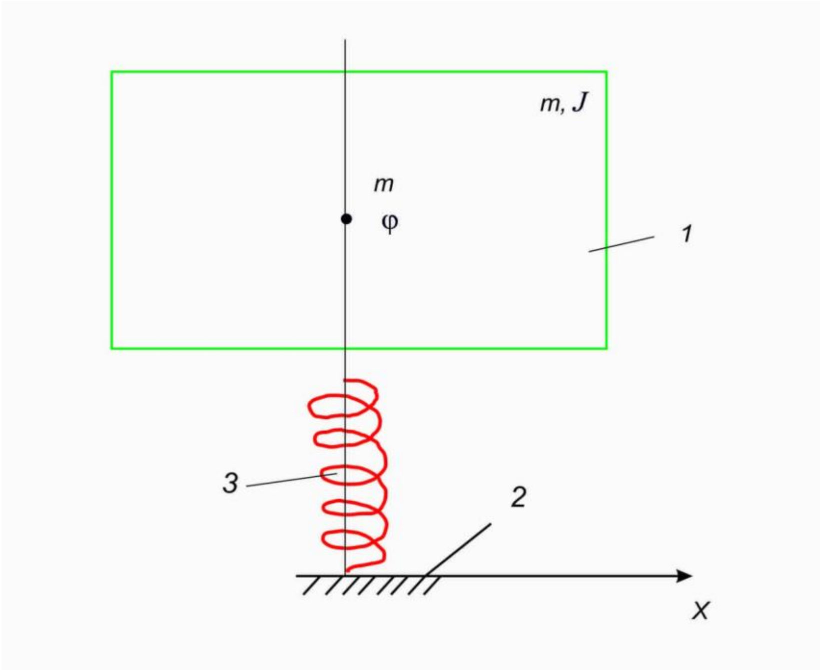

Figure 1. Calculation scheme: 1-solid, 2-base, 3-spring

Equations (19) can be transformed in the following sequence

$$\left.\begin{array}{l} \ddot{f}_1-\ddot{f}_{01}+\Omega^2\left[f_1-f_{01}-\int\limits_{-\infty}^{t}R(t-\tau)(f_1-f_{01})d\tau\right]=\mu F(f_{01},f_{02},t), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ddot{f}_k-\ddot{f}_{k-1}+\Omega^2\left[f_k-f_{k-1}-\int\limits_{-\infty}^{t}R(t-\tau)(f_k-f_{k-1})d\tau\right]=\mu F(f_{1,k-1},f_{2,k-1},t). \end{array}\right\} \quad (20)$$

3. Results and analysis

Estimates of periodic solutions of equations (20) are given in [12, p.8].

When solving specific tasks, the following mechanical characteristics of the body were adopted: *m=1; l=0.5; I=0.1*. The relaxation core is taken as

$$R(t-s)=\frac{Ae^{-\beta(t-s)}}{(t-s)^{1-\alpha}}.$$

Two variants of kernel parameters (21) are considered:

1) $\alpha=0{,}1;\quad A=0.078;\quad \beta=0.05$

2) $\alpha=0{,}1;\quad A=0.048;\quad \beta=0.05$

The results found at low and high viscosities are qualitatively the same, differing only by significantly large values of amplitudes at low viscosity. Therefore, the analysis of the obtained solutions is given only for high viscosity.

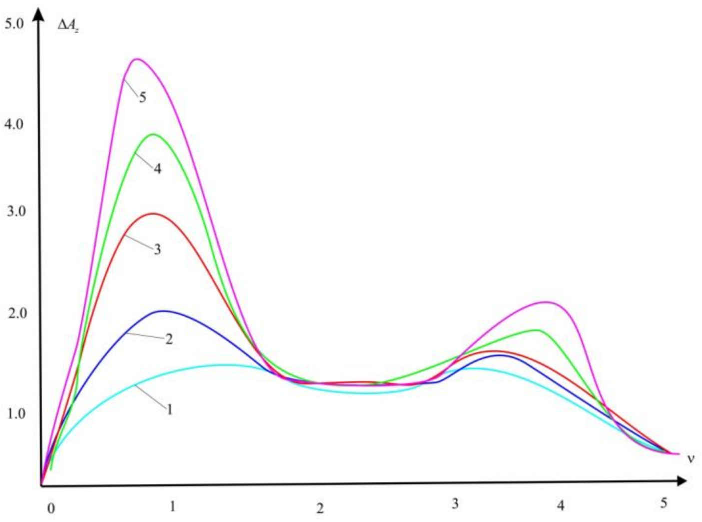

Figure 2. Change in the amplitude of displacements from the frequency of external loads: 1. A=0.078; 2. A=0.048; 3. A=0.030; 4. A=0.020; A=0.015

Figure 2 shows the results of calculations for nonlinear problems.

According to numerical results, the amplitude value for a nonlinear problem is 3-4% greater than for a linear one.

4. Conclusions

The paper presents a method for solving the problem of nonlinear oscillations of a mechanical system with two degrees of freedom. Based on the numerical results obtained, it is found that the amplitude value for a nonlinear problem is 3-4% greater than for a linear one.

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