BIRINCHI TARTIBLI BIR JINSLI DIFFERENSIAL TENGLAMALAR

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Annotatsiya: *Bugungi kunda matematikaning differensial tenglamalar bo‘limi juda rivojlanmoqda. Ta’lim sohasida esa alohida e’tibor qaratilmoqda. Shu bilan birgalikda differensial tenglamalar orqali ko‘pgina masalalar o‘z yechimini topmoqda. Ushbu maqolada birinchi tartibli bir jinsli differensial tenglamalar haqida ma’lumot va bunday ko‘rinishdagi tenglamalarni qanday ishlash bir nechta misol yordamida ko‘rsatib o‘tilgan.*

Kalit so‘zlar: *differensial tenglamalar, bir jinsli differensial tenglamalar, uzluksiz funksiyalar, umumiy yechim, o‘zgaruvchilari ajraladigan differensial tenglamalar, integral, birinchi tartibli bir jinsli differensial tenglamalar.*

Annotation: *Today, the branch of differential equations of mathematics is developing very much. Special attention is being paid in the field of education. At the same time, many problems are being solved through differential equations. This article provides information on first-order homogeneous differential equations and how to work with equations of this form using several examples.*

Key words: *differential equations, homogeneous differential equations. continuous functions, general solution, differential equations with separable variables, integral, homogeneous differential equations of the first order.*

Ta’rif. Ushbu $y' = f(x, y)$ tenglama bir jinsli tenglama deyiladi, agar f(x,y) funksiyani argumentlarining nisbati orqali ifodalash mumkin bo‘lsa, ya’ni:

$$y' = f\left(1, \frac{y}{x}\right)$$

Yoki

$$y' = f\left(\frac{y}{x}\right) \qquad (1)$$

Misol uchun, $(xy - y^2)dx - (x^2 - 2xy)dy = 0$

Tenglama bir jinsli tenglamadir. Haqiqatdan ham bu ifodani ushbu ko‘rinishda yozish mumkin:

$$\frac{dy}{dx} = \frac{xy - y^2}{x^2 - 2xy} = \frac{\frac{y}{x} - \left(\frac{y}{x}\right)^2}{1 - 2\frac{y}{x}}$$

(1) Tenglamada $x \neq 0, f\left(\frac{y}{x}\right)$ funksiya x va y ning barcha qaralayotgan qiymatlarida uzluksizdir. Bu tenglama

$$\frac{y}{x} = u, \qquad y = ux, \qquad y' = xu' + u$$

O‘rniga qo‘yish bilan o‘zgaruvchilari ajraladigan tenglamaga keltiriladi.

$$xu' + u = \varphi(u)$$

Yoki
$$x\frac{du}{dx} = \varphi(u) - u$$

Bundan quyidagi o‘zgaruvchilari ajralgan tenglama hosil bo‘ladi:

$$\frac{du}{\varphi(u) - u} = \frac{dx}{x}$$

1. Misol. $xdy - ydx = ydy$

$$(x - y)dy = ydx$$

$$\left(1 - \frac{y}{x}\right)\frac{dy}{dx} = \frac{y}{x}$$

Quyidagicha belgilash kiritamiz:

$$\frac{y}{x} = u, \quad y = ux, \quad y' = xu' + u$$

Yoki
$$\frac{dy}{dx} = x\frac{du}{dx} + u$$

$$(1-u)\left(x\frac{du}{dx} + u\right) = u$$

$$\frac{1}{u^2}du - \frac{1}{u}du = \frac{dx}{x}$$

Ushbu birinchi tartibli bir jinsli differensial tenglamani ikkala tomonini integrallab:

$$\int \frac{1}{u^2}du - \int \frac{1}{u}du = \int \frac{dx}{x}$$

Quyidagiga ega bo‘lamiz:

$$-\frac{1}{u} - \ln u = \ln x + \ln C$$

$\frac{y}{x} = u$ ifodani o‘rniga qaytarib olib kelib qo‘ysak, quyidagi yechimga ega bo‘lamiz:

$$-\frac{x}{y} - \ln\frac{y}{x} = \ln(xC)$$

Tenglama umumiy yechimga ega bo‘ldi.

2. Misol. $y' = \frac{x+y}{x-y}$ tenglamani yeching.

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

Tenglikning o‘ng tarafini x ga bo‘lamiz:

$$\frac{dy}{dx} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$$

$$\frac{y}{x} = u, \quad y = ux, \quad y' = xu' + u$$

yoki $\quad \frac{dy}{dx} = x\frac{du}{dx} + u \quad \Rightarrow x\frac{du}{dx} + u = \frac{1+u}{1-u} \Rightarrow \quad x\frac{du}{dx} = \frac{1+u}{1-u} - u$

$$x\frac{du}{dx} = \frac{1+u-u+u^2}{1-u}$$

$$\frac{1-u}{1+u^2}du = \frac{dx}{x}$$

Tenglikning ikkala tarafini integrallaymiz:

$$arctgu - \frac{1}{2}ln|1+u^2| = lnx + lnC$$

$$arctg\frac{y}{x} - \frac{1}{2}ln\left|1+\left(\frac{y}{x}\right)^2\right| = lnx + lnC$$

Tenglama quyidagi umumiy yechimga ega bo‘ladi:

$$arctg\frac{y}{x} - \frac{1}{2}ln\left|1+\left(\frac{y}{x}\right)^2\right| = lnxC$$

3. Misol. $xy' = 3y - x$ tenglamani yeching.

$$x\frac{dy}{dx} = 3y - x$$

Tenglikning ikkala tarafini ham x ga bo‘lamiz:

$$\frac{dy}{dx} = 3\frac{y}{x} - 1$$

Quyidagicha belgilash kiritamiz:

$$\frac{y}{x} = u, \qquad y = ux, \qquad y' = xu' + u$$

Yoki $$\frac{dy}{dx} = x\frac{du}{dx} + u$$

$$x\frac{du}{dx} + u = 3u - 1$$

$$x\frac{du}{dx} = 3u - u - 1$$

$$x\frac{du}{dx} = 2u - 1$$

$$\frac{du}{2u-1}=\frac{dx}{x}$$

Tenglikning ikkala tarafini ham integrallaymiz va quyidagi natijani hosil qilamiz:

$$\frac{1}{2}ln|2u-1|=lnx+lnC$$

$$ln|2u-1|^{\frac{1}{2}}=lnx+lnC$$

Tenglikning ikkala tarafini ham logarifmlaymiz:

$$|2u-1|^{\frac{1}{2}}=|xC|$$

$\frac{y}{x}=u$ belgilashni o‘rniga qo‘yamiz:

$$\left|2\frac{y}{x}-1\right|^{\frac{1}{2}}=xC$$

Ko‘rinishdagi umumiy yechimga ega bo‘lamiz.

RO‘YXATI

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