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MEXANIKA MASALALARINI GRAFIK USULDA WOLFRAM MATHEMATICA DASTURI ASOSIDA YECHISH

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ANNOTATSIYA

Ushbu ilmiy maqolada klassik mexanikaning asosiy bo‘limlariga oid masalalarni Wolfram Mathematica dasturi yordamida grafik va sonli usullarda yechish hamda tahlil qilish masalalari ko‘rib chiqilgan. Mexanik jarayonlarni matematik modellashtirish ko‘pincha murakkab differensial tenglamalar va analitik yechimlar bilan bog‘liq bo‘lib, bunday holatlarda grafik va sonli yondashuvlar muhim ahamiyat kasb etadi. Maqolada otilgan jism harakati, havo qarshiligi bilan harakat, qiya sirt bo‘yicha sirpanish, oddiy mayatnik, garmonik tebranuvchi, ikki massa-prujina tizimi, markaziy kuch maydonida harakat, shuningdek impuls va energiya saqlanish qonunlari misollar orqali tahlil qilingan. Har bir mexanik model uchun tegishli matematik tenglamalar tuzilib, Wolfram Mathematica muhitida ularning grafik tasvirlari, faza portretlari va sonli yechimlari olingan. Olingan natijalar mexanik jarayonlarning fizik mohiyatini chuqurroq tushunishga, parametrlarning o‘zgarishi harakat xarakteriga qanday ta’sir ko‘rsatishini aniqlashga imkon beradi. Tadqiqot natijalari Wolfram Mathematica dasturidan mexanika fanini o‘qitishda, laboratoriya

mashg‘ulotlarida hamda ilmiy tadqiqotlarda samarali foydalanish mumkinligini ko‘rsatadi.

Kalit so‘zlar: *Klassik mexanika, Wolfram Mathematica, otilgan jism harakati, grafik va raqamli usullar, havo qarshiligi, garmonik tebranuvchi, markaziy kuch maydoni*

ABSTRACT

This scientific article considers the solution and analysis of problems from the main sections of classical mechanics using graphical and numerical methods implemented in the Wolfram Mathematica software. The mathematical modeling of mechanical processes is often associated with complex differential equations and analytical solutions; therefore, graphical and numerical approaches play a crucial role in understanding such systems. The paper analyzes projectile motion, motion with air resistance, sliding on an inclined plane, the simple pendulum, the harmonic oscillator, a two mass-spring system, motion in central force fields, as well as the laws of conservation of momentum and energy through illustrative examples. For each mechanical model, the corresponding mathematical equations are formulated and their graphical representations, phase portraits, and numerical solutions are obtained in the Wolfram Mathematica environment. The results clearly demonstrate the physical nature of mechanical processes and show how variations in system parameters influence the character of motion. The study confirms that Wolfram Mathematica is an effective and versatile tool for visualizing, modeling, and analyzing both linear and nonlinear mechanical systems. The obtained results indicate that this software can be efficiently used in teaching classical mechanics, conducting laboratory work, and performing scientific research.

KIRISH

Mexanika jismlarning harakati, kuchlarning ta’siri ostida ularning o‘zgarishi va dinamik xususiyatlarini o‘rganuvchi fundamental fandir. Ushbu jarayonlar matematik

modellar orqali ifodalanadi. Bunday modellar ko‘pincha differensial tenglamalar, integral hisoblash, vektor funksiyalar, faza fazosida tahlil kabi murakkab matematik apparatni talab qiladi.

ADABIYOTLAR TAHLILI VA METODOLOGIYA

Wolfram Mathematica dasturi murakkab mexanik jarayonlarni modellashtirish, vizualizatsiya qilish va sonli yechimlar olish uchun eng qulay ilmiy dasturiy vositalardan biridir. Mathematica yordamida mexanik tizimlarning grafik modeli, faza trayektoriyalari, kuch vektor maydonlari, energiyaning vaqt bo‘yicha o‘zgarishi, tebranishlar va murakkab nochiziqli tizimlarning yechimlari oson aniqlanadi [1-4].

Grafik metod mexanika masalalarini o‘rganishda juda muhim. Chunki u jarayonni ko‘z oldiga keltirish, parametrlarning o‘zgarishi natijasida harakat shakllarining qanday o‘zgarishini kuzatish, murakkab tenglamalarning fizik mazmunini anglashga yordam beradi. Aynan shuning uchun, ushbu maqolada grafik yechimlar asosiy o‘rinda turadi [5-6].

NATIJALAR

Mathematica mexanikada qo‘llanishi mumkin bo‘lgan kuchli vositalarga ega.

Ulardan ayrimlari:

-Symbolik hisoblash.

- Sonli yechim: NDSolve.

- Grafik chizish: Plot, ParametricPlot, VectorPlot, StreamPlot.

- Interaktiv modellar: Manipulate.

- Dinamik animatsiya: Animate.

- Jadvallar: Table, Grid, Dataset.

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Bu imkoniyatlar mexanik jarayonlarni chuqur va aniq o‘rganish uchun sharoit yaratadi. Masalan, differensial tenglama analitik yechimi mavjud bo‘lmagan holatlarda NDSolve yordamida raqamli yechim olinadi.

Otilgan jism harakati

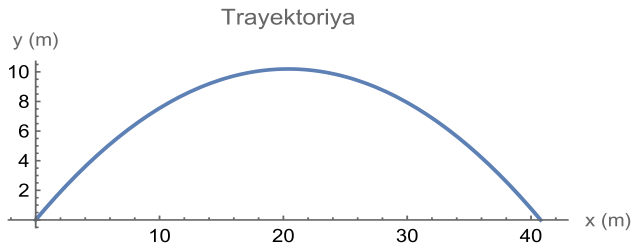
Otilgan jism harakatida jism boshlang‘ich tezlik v_0 va burchak θ ostida otib yuboriladi. Analitik model quyidagicha:

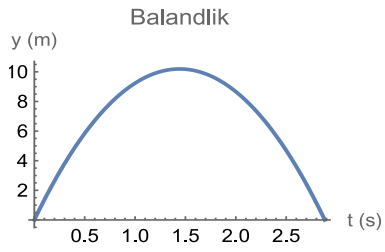
$x(t)=v_0 \cos(\theta)\, t,$

$y(t)=v_0 \sin(\theta)\, t - (1/2)\, g\, t^2.$

Traektoriya parabola hosil qiladi.

```
ClearAll["Global`*"];
v0=20; (*m/s*)
thetaDeg=45; (*degree*)
g=9.81;
theta=thetaDeg Degree;
x[t_]:=v0 Cos[theta] t;
y[t_]:=v0 Sin[theta] t-1/2 g t^2;
Tflight=2 v0 Sin[theta]/g;
Range=v0 Cos[theta] Tflight;
Hmax=v0^2 Sin[theta]^2/(2 g);
Print["Flight time T = ",N[Tflight]," s"];
Print["Range R = ",N[Range]," m"];
Print["Max height Hmax = ",N[Hmax]," m"];
GraphicsRow[{ParametricPlot[{x[t],y[t]},{t,0,Tflight},AxesLabel->{"x (m)","y (m)"},PlotRange->All,AspectRatio->1/3,PlotLabel->"Trayektoriya"],Plot[y[t],{t,0,Tflight},AxesLabel->{"t (s)","y (m)"},PlotLabel->"Balandlik"]}]
```

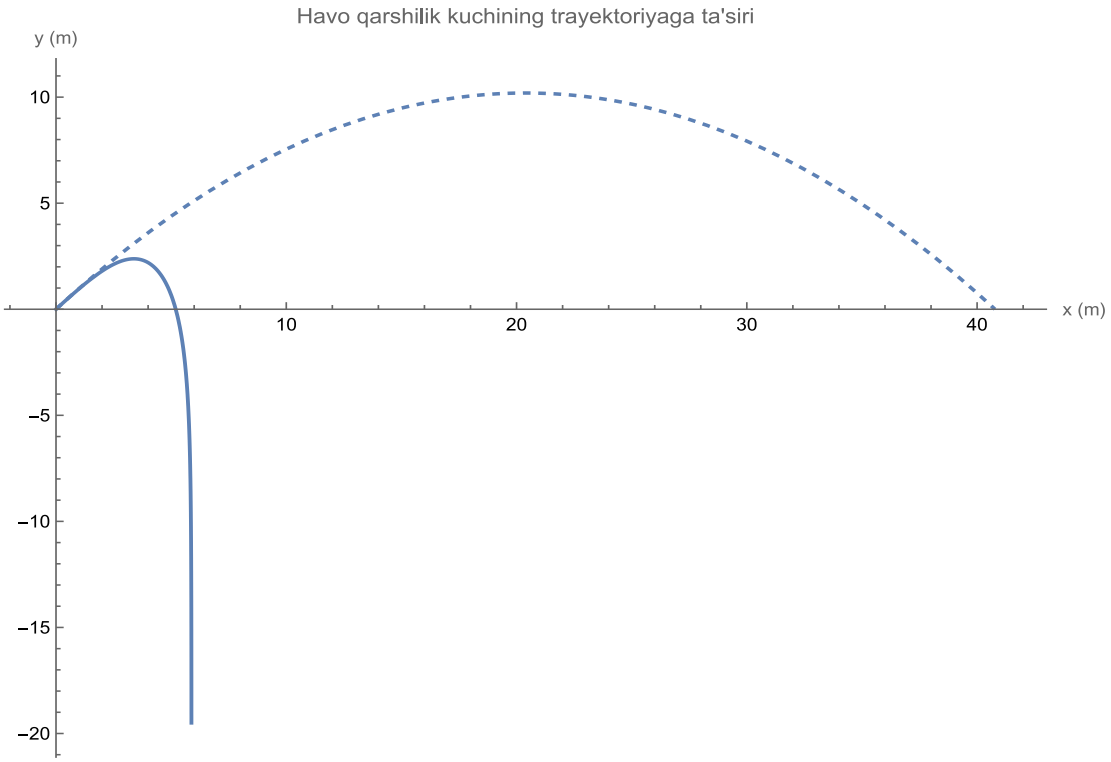




Havo qarshiligi bilan otilgan jism harakati

Havo qarshiligi mavjud bo‘lgan holda harakat tenglamalari nochiziqli bo‘lib ketadi. Qarshilik kuchi $-c\,v^2$ ko‘rinishida olinadi. Bu tizimni faqat sonli usulda yechish mumkin. NDSolve yordamida x(t) va y(t) funksiyalari olinadi. Grafik chizmalar havo qarshiligi trayektoriyani qanday qisqartirishini aniq ko‘rsatadi.

```
ClearAll["Global`*"];
m=0.15; (*kg*)
cd=0.05; (*drag coefficient (effective)*)
v0=20;thetaDeg=45;
g=9.81;
theta=thetaDeg Degree;
(*System:x’’=-(cd/m) v x’,y’’=-g-(cd/m) v y’ where v=Sqrt[x’^2+y’^2]*)
tmax=5;
sol=NDSolve[{x’’[t]==-(cd/m) Sqrt[x’[t]^2+y’[t]^2] x’[t],y’’[t]==-g-(cd/m) Sqrt[x’[t]^2+y’[t]^2] y’[t],x[0]==0,y[0]==0,x’[0]==v0 Cos[theta],y’[0]==v0 Sin[theta]},{x,y},{t,0,tmax},MaxSteps->20000,AccuracyGoal->8];
traj=ParametricPlot[Evaluate[{x[t],y[t]}/. sol],{t,0,tmax},AxesLabel->{"x (m)","y (m)"},PlotRange->All,PlotLabel->"Havo qarshilik kuchining trayektoriyaga ta’siri"];
(*Compare with no-drag parabola*)
par=ParametricPlot[{v0 Cos[theta] t,v0 Sin[theta] t-1/2 g t^2},{t,0,2 v0 Sin[theta]/g},PlotStyle->{Dashed},PlotRange->All];
Show[traj,par,ImageSize->Large]
```



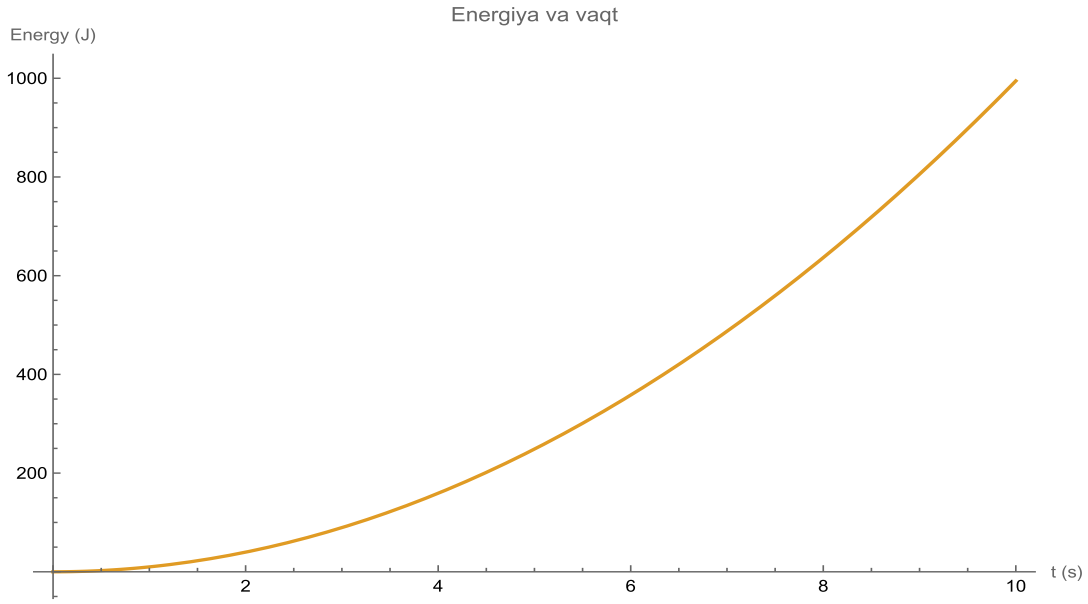
Qiya sirt bo‘yicha harakat

Qiya sirt mexanikada asosiy modeldir. Jism sirpanayotganda unga og‘irlik kuchi, normal reaksiya va ishqalanish kuchi ta’sir qiladi. Harakat energiya tenglamasi orqali ham, kuchlarning proyeksiyasi orqali ham o‘rganiladi. Mathematica yordamida jismning yo‘li, tezligi, tezlanishi, energiyasi vaqt bo‘yicha grafik tarzda chiqariladi.

```
ClearAll["Global`*"];
m=1;g=9.81;
alphaDeg=30;mu=0.1;v0=0;
alpha=alphaDeg Degree;
a=g Sin[alpha]-mu g Cos[alpha]; (*acceleration along plane*)
s[t_]:=1/2 a t^2+v0 t; (*displacement along incline*)
v[t_]:=D[s[t],t];
Kinetic[t_]:=1/2 m v[t]^2;
Potential[t_]:=m g s[t] Sin[alpha]; (*height=s Sin alpha*)
TotalE[t_]:=Kinetic[t]+Potential[t];
Print["Acceleration a = ",N[a]," m/s^2"];
```

GraphicsRow[{Plot[s[t],{t,0,10},AxesLabel->{"t (s)","s (m)"},PlotLabel->"Displacement vs t"],Plot[v[t],{t,0,10},AxesLabel->{"t (s)","v (m/s)"},PlotLabel->"Velocity vs t"]},ImageSize->Large]

Plot[{Kinetic[t],Potential[t],TotalE[t]},{t,0,10},PlotLegends->{"Kinetic","Potential","Total"},AxesLabel->{"t (s)","Energy (J)"},PlotLabel->"Energies vs t",ImageSize->Large]



Oddiy mayatnik

Oddiy mayatnik nochiziqli tizim bo‘lib, tenglama θ’’ + (g/l) sin θ = 0 ko‘rinishida bo‘ladi. Kichik burchaklarda chiziqli model qo‘llanadi, katta burchaklarda esa to‘liq nochiziqli yechim talab qilinadi. Mathematica mayatnikning faza portretlari, energiya funksiyalari, tebranish davrini aniqlash imkonini beradi.

ClearAll["Global`*"];

g=9.81;l=1;

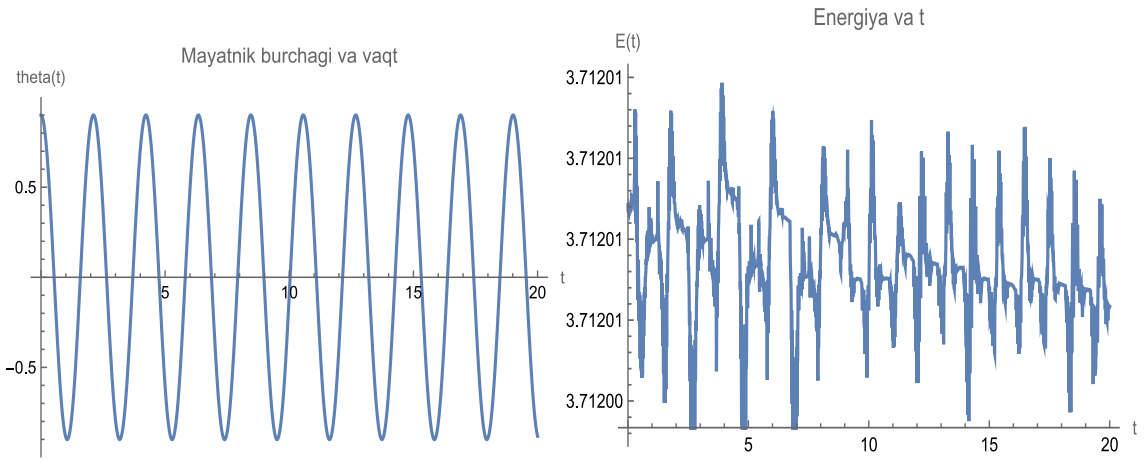
theta0=0.9;omega0=0;tmax=20;

solPend=NDSolve[{theta’’[t]+(g/l) Sin[theta[t]]==0,theta[0]==theta0,theta’[0]==omega0},theta,{t,0,tmax},MaxSteps->Infinity];

(*theta(t)*)

Plot[Evaluate[theta[t]/. solPend],{t,0,tmax},AxesLabel->{"t","theta(t)"},PlotLabel->"Mayatnik burchagi va vaqt"]

```
(*Phase portrait:theta vs omega*)
phase=ParametricPlot[Evaluate[{theta[t],theta'[t]}/.
solPend],{t,0,tmax},AxesLabel->{"theta","omega"},PlotRange->All,PlotLabel-
>"Phase portrait"];
(*Energy*)
Energy[t_]:=1/2 l^2 (theta'[t])^2+g l (1-Cos[theta[t]]);
Plot[Evaluate[Energy[t]/.                    solPend],{t,0,tmax},AxesLabel-
>{"t","E(t)"},PlotLabel->"Energiya va t"]
```

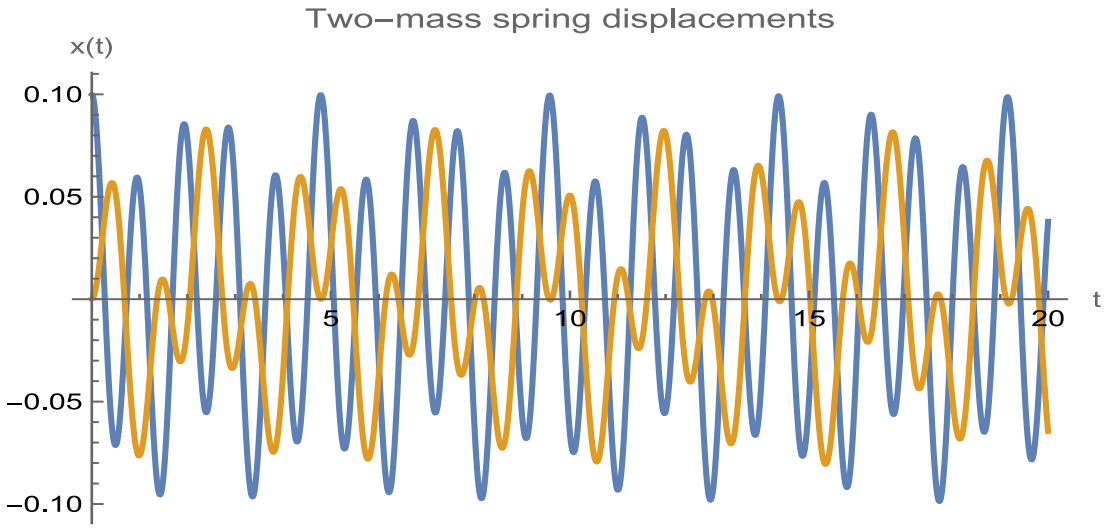


Ikki massa — prujina tizimi

Ikki massa bir-biriga prujina bilan ulanib, murakkab garmonik tizim hosil qiladi. Bu tizimda energiya almashinuvi, rezonans, normal modalar mavjud. Mathematica yordamida $x_1(t)$ va $x_2(t)$ yechimlari grafik chiziladi.

```
ClearAll["Global`*"];
m1=1.2;m2=0.8;
v1i=2;v2i=0; (*initial velocities*)
(*Elastic collision formulas*)
v1f=((m1-m2) v1i+2 m2 v2i)/(m1+m2);
v2f=(2 m1 v1i+(m2-m1) v2i)/(m1+m2);
Print["After elastic collision: v1f = ",N[v1f]," , v2f = ",N[v2f]];
(*Check conservation*)
Print["Initial momentum: ",N[m1 v1i+m2 v2i]];
```

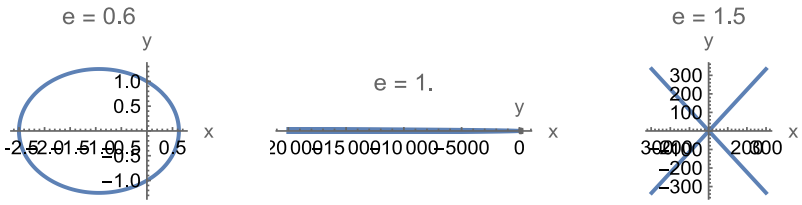
```
Print["Final momentum: ",N[m1 v1f+m2 v2f]];
Print["Initial KE: ",N[1/2 m1 v1i^2+1/2 m2 v2i^2]];
Print["Final KE: ",N[1/2 m1 v1f^2+1/2 m2 v2f^2]];
```



Markaziy kuch maydonida harakat

Gravitatsion yoki elektrostatik markaziy kuch ostida jismning harakati r(θ)=p/(1+e cos θ) formulasi bilan ifodalanadi. Mathematica yordamida planetar orbitaning barcha turlari - elliptik, parabolik va giperbolik trayektoriyalar chiziladi.

```
p=1; (*semi-latus rectum*)
eList={0.6,1.0,1.5}; (*eccentricities:ellipse,parabola,hyperbola*)
orbits=Table[ParametricPlot[Evaluate[{(p/(1+e Cos[theta])) Cos[theta],(p/(1+e Cos[theta])) Sin[theta]}],{theta,-Pi+0.01,Pi-0.01},PlotRange->All,AxesLabel->{"x","y"},PlotLabel->"e = "<>ToString[e]],{e,eList}];
GraphicsRow[orbits,ImageSize->Large]
```



Garmonik tebranuvchi

Garmonik tebranuvchi x’’ + ω² x = 0 tebranish tizimi bo‘lib, energiya saqlanadi va faza portreti ellips ko‘rinishida bo‘ladi. Mathematica uning dinamikasini to‘liq ochib beradi.

```
ClearAll["Global`*"];
omega=2 Pi;x0=1;v0=0;tmax=4;
x[t_]:=x0 Cos[omega t]+(v0/omega) Sin[omega t];
v[t_]:=D[x[t],t];
Plot[x[t],{t,0,tmax},AxesLabel->{"t","x(t)"},PlotLabel->"Garmonik tebranish"]
ParametricPlot[{x[t],v[t]},{t,0,tmax},AxesLabel->{"x","v"},PlotLabel->"Phase portrait (ellipse)"]
(*Energy*)
E=1/2 (v[t])^2+1/2 omega^2 x[t]^2;
Simplify[E]
```

Jadvallar va sonli tahlil

Mathematica natijalarni jadval ko‘rinishida chiqarishi orqali mexanik jarayonlarni raqamli tahlil qilishni osonlashtiradi. Bu natijalar laboratoriya ishlari uchun juda qulay.

```
ClearAll["Global`*"];
v0=20;g=9.81;
angles=Range[10,80,10] Degree;
ranges=Table[R=v0^2 Sin[2 theta]/g;
{N[theta/Degree],N[R]},{theta,angles}];
ds=Dataset[Map[AssociationThread[{"AngleDeg","Range_m"}->#]&,ranges]];ds
```

AngleDeg	Range_m
10.0	13.9458
20.0	26.2095
30.0	35.3119
40.0	40.1553
50.0	40.1553
60.0	35.3119
70.0	26.2095
80.0	13.9458

MUHOKAMA

Ushbu tadqiqotda klassik mexanikaning asosiy masalalari Wolfram Mathematica dasturi yordamida grafik va raqamli usullarda tahlil qilindi. Olingan natijalar Mathematica analitik yechimlarni tasdiqlash bilan birga, nochiziqli va murakkab mexanik jarayonlarning fizik mazmunini aniq ko‘rgazmali ko‘rinishda ifodalash imkonini berishini ko‘rsatdi.

Otilgan jism harakati, qiya sirt va garmonik tebranuvchi misollarida grafik yechimlar nazariy formulalar bilan mos kelishi aniqlandi. Havo qarshiligi va oddiy mayatnik kabi nochiziqli tizimlarda esa faqat sonli usullar orqali yechim olinib, faza portretlari va trayektoriyalar yordamida tizim dinamikasi chuqurroq tushuntirildi. Markaziy kuch maydonlari va energiya saqlanishiga oid modellar mexanik qonunlarning umumiyligini grafik tarzda namoyon etdi.

Umuman olganda, Wolfram Mathematica mexanika masalalarini o‘rganish va o‘qitishda samarali vosita bo‘lib, u nazariy bilimlarni amaliy va ko‘rgazmali tahlil bilan mustahkamlash imkonini beradi.

XULOSA

Mazkur maqolada klassik mexanikaning asosiy bo‘limlariga oid masalalar Wolfram Mathematica dasturi yordamida grafik va raqamli usullarda tahlil qilindi. Olingan natijalar ushbu dastur mexanik jarayonlarni modellashtirish, nolinear tizimlarni o‘rganish va fizik qonunlarni vizual tarzda tushuntirishda samarali vosita ekanligini ko‘rsatdi. Grafik yondashuv murakkab harakatlarni intuitiv anglashga yordam berib, analitik va sonli yechimlarni o‘zaro solishtirish imkonini berdi. Tadqiqot

natijalari Wolfram Mathematica dasturidan mexanika fanini o‘qitish va ilmiy tadqiqotlarda keng foydalanish mumkinligini tasdiqlaydi.

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